Capacitance Parameters of Coupled Elliptic Microstrip Disks in Layered Anisotropic Media

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Abstract —**An algorithm is provided to calculate the modal capacitances and the gap capacitance of coupled microstrip elliptic disks embedded in layered media with dielectric anisotropy. The capacitance parameters determined are useful in accounting for coupling effects in the design of interacting lumped microstrip components of elliptic shape.**

I. Introduction

When several printed microstrip conductors are involved in the same circuit, the coupling effects between different conductors cannot be neglected. In this case, the modal capacitances or the capacitance matrix coefficients are valuable parameters in describing the behavior of the circuit in a reasonable way [1], [2]. Resonators consisting of two coupled microstrip patches were analyzed by Uzunoglu and Katechi [3] in terms of a circuit model where the gap capacitance between conductor patches was used to account for coupling. This model is simple and it provides reliable results when compared with experimental data.

In this paper, we present an algorithm to calculate modal capacitances and the gap capacitance of two symmetrically coupled microstrip elliptic patches. The calculation of the gap capacitance of coupled circular patches has already been reported [3], [4]. Here, we introduce the eccentricity of the ellipses as an additional design parameter because it is known that elliptic patches have certain advantages over circular patches for resonator and antenna applications [5]. In the algorithm built, conductor elliptic patches are embedded in a multilayered substrate with dielectric anisotropy. Variational techniques in the spectral domain have been employed to calculate lower bounds for the modal capacitances of the two-conductor configuration. The value of the gap capacitance defined in [3] is easily obtained in terms of the odd-mode capacitance and the capacitance of an isolated elliptic plate [5].

II. Spectral-Domain Computation of the Modal Capacitances

Fig. 1(a) stands for the cross section of a lossless multidielectric stratified medium with dielectric anisotropy. Two coplanar elliptic conductor plates with equal dimensions lie on the surface of the Mth dielectric layer of the stratified configuration. Conductors are assumed to be lossless and infinitely thin. The position of the ellipses with respect to the Cartesian coordinate system chosen is shown in Fig. 1(b). The analysis carried out in this paper not only includes the case $a > b$ (elliptic plates with aligned major axes), which is shown in Fig. 1(b); it also includes the cases $a = b$ (circular plates) and $a < b$ (elliptic plates with aligned minor axes). The $x = 0$ plane in parts (a) and (b) of Fig. 1 is assumed to be a geometrical symmetry plane for the problem. Owing to this, the permittivity tensor of the anisotropic dielectrics must be of the form

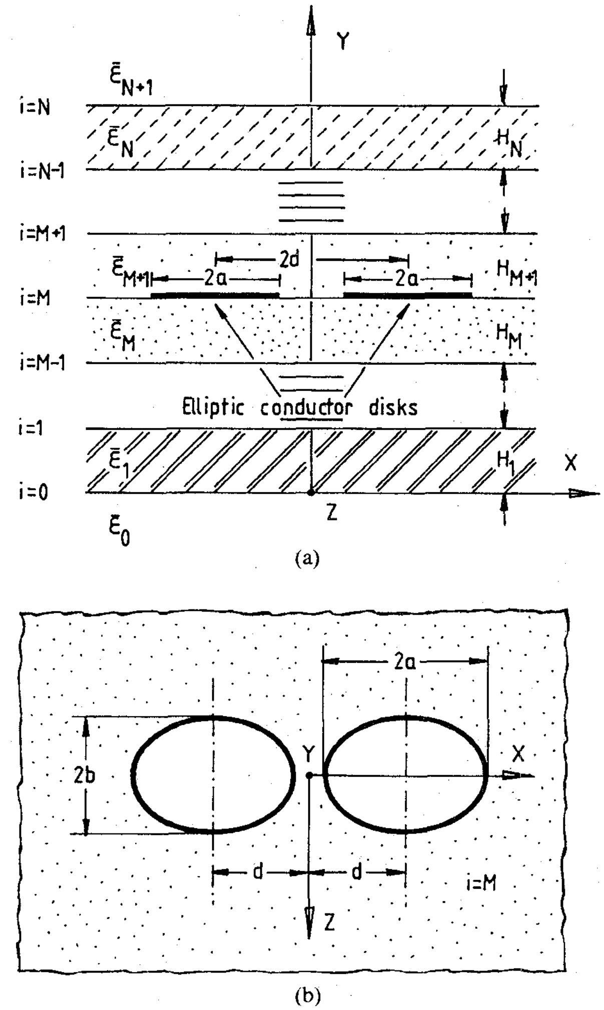

Fig. 1. (a) Cross section of a multilayered anisotropic substrate in which the elliptic disks of Fig. 1(b) are embedded. (b) Geometry for coupled elliptic disks ($a > b$).

$$\bar{\epsilon}_i = \epsilon_0 \begin{bmatrix} \epsilon^*_{11,i} & 0 & 0 \\ 0 & \epsilon^*_{22,i} & \epsilon^*_{23,i} \\ 0 & \epsilon^*_{23,i} & \epsilon^*_{33,i} \end{bmatrix} \qquad (i = 0, \cdots, N+1). \tag{1}$$

Boundary interfaces $i = 0$ and $i = N$ in Fig. 1(a) can independently represent electric walls, magnetic walls, or open boundaries extending to infinity.

Since the coupling is symmetrical, even and odd modes of excitation exist. Therefore, the capacitance matrix of the two-conductor configuration is completely specified in terms of the modal capacitances C_e and C_o. The gap capacitance between the conductor plates defined in [3] can be expressed in terms of the odd capacitance as

$$C_g = \frac{1}{2}(C_o - C). \tag{2}$$

Here C stands for the capacitance of one of the elliptic plates in

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the absence of the other elliptic plate. This capacitance value can be obtained by using the algorithm proposed in [5].

When a two-dimensional Fourier transform is performed from the coordinates x and z defined in parts (a) and (b) of Fig. 1 to the spectral variables α and β, the electric energy associated with each mode in the two-conductor configuration can be expressed as [5], [6]

$$U_{e,o} = \frac{1}{8\pi^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} G(\alpha,\beta)|\tilde{\rho}_{e,o}(\alpha,\beta)|^2\, d\alpha\, d\beta \tag{3}$$

where $G(\alpha,\beta)$ is the spectral Green's function defined in [6] and $\tilde{\rho}_{e,o}(\alpha,\beta)$ stand for the Fourier transforms of the charge density on both elliptic plates in each mode. If an adequate approximation is used for the charge density on the elliptic plates in each mode, the stationary properties of (3) make it possible to obtain an accurate value of the modal energies $U_{e,o}$ [5], [6]. Since a symmetry plane exists in the configuration proposed, we only need to approximate the charge density on one conductor plate to calculate $U_{e,o}$. This conductor plate has been chosen to be the one corresponding to $x > 0$ in Fig. 1(b). To obtain the transform of the charge density on the mentioned plate in closed form, a local coordinate transformation specially adapted to the elliptic geometry of the plate is carried out [5]. This coordinate transformation can be written as

$$\begin{aligned} x - d &= r'\cos\phi' \\ z &= (b/a)r'\sin\phi'. \end{aligned} \tag{4}$$

In [5] it was assumed that the charge density on the conductor elliptic plate was not a function of the local ϕ' coordinate defined in (4). When we analyze coupled elliptic plates, the error obtained by neglecting the charge density dependence on the ϕ' coordinate seems to be much more important, owing to coupling effects. In the odd mode, the charge on each conductor plate is concentrated in the region closest to the other plate, whereas in the even mode, the charge is concentrated in the region farthest from the other plate. To account for these charge displacements in a reasonable way, the charge density approximation on the elliptic plates should include the dependence on the ϕ' coordinate.

For substrates having the type of anisotropy assumed in (1), the plane $z = 0$ in Fig. 1(b) is a magnetic wall. Owing to this, the modal charge densities on the ellipse on the right in Fig. 1(b) must be such that $\rho_{e,o}(r',\phi') = \rho_{e,o}(r',-\phi')$, where r' and ϕ' are defined in (4). To satisfy this condition, we have approximated the charge density on that elliptic plate by using trial function expansions of the form

$$\rho_{e,o}(r',\phi') = \sum_{m=0}^{p}\sum_{n=0}^{q} C_{mn}^{e,o}\rho_{mn}(r',\phi') \tag{5a}$$

where

$$\rho_{mn}(r',\phi') = \rho_n(r')\cos(m\phi') \qquad (m = 0,\cdots,p;\ n = 0,\cdots,q). \tag{5b}$$

The opposite charge displacements on the plates in each mode can be accurately accounted for by using an adequate number of cosine functions on the ϕ' coordinate in (5a).

Following [5], we assume that the functions $\rho_n(r')$ are given by

$$\rho_n(r') = \frac{T_{2n}(r'/a)}{\left(1-(r'/a)^2\right)^{1/2}} \qquad (n = 0,\cdots,q). \tag{6}$$

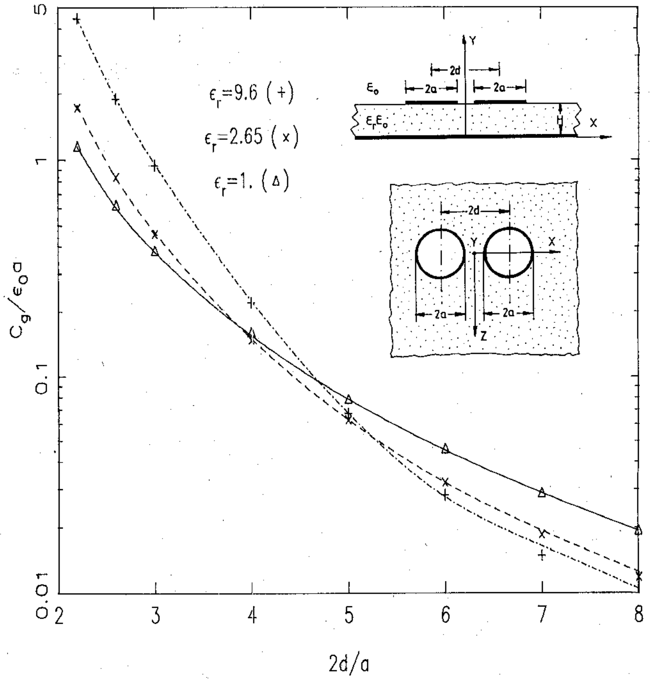

Fig. 2. Gap capacitance of coupled circular microstrip disks as a function of the normalized distance between the disks $2d/a$ ($a/H = 1.0$ is used in all the graphs). Comparison with the results reported in [4] $(+,\times,\Delta)$ is provided.

In this expression, $T_{2n}(x)$ are Chebyshev polynomials of the first kind.

It can be seen that all the functions defined in (6) account for the charge density singularity at the edge of the elliptic plate. The number of the functions $\rho_n(r')$ necessary to provide an accurate approximation of the charge density dependence on the r' variable must be higher as the plate surface/substrate thickness ratio increases [5].

When the modal charge densities on the right plate in Fig. 1(a) are assumed to be of the form proposed in (5a) and (5b), the analytical determination of the spectral functions $\tilde{\rho}_{e,o}(\alpha,\beta)$ can be considerably simplified by using a variable transformation in the two-dimensional Fourier domain [5]. This variable transformation is given by

$$\begin{aligned} \alpha &= \gamma\cos\Omega \\ \beta &= (a/b)\gamma\sin\Omega. \end{aligned} \tag{7}$$

In terms of the new variables γ and Ω the modal charge density Fourier transforms can be expressed as

$$\tilde{\rho}_{e,o}(\gamma,\Omega) = \sum_{m=0}^{p}\sum_{n=0}^{q} C_{mn}^{e,o}\tilde{\rho}_{mn}^{e,o}(\gamma,\Omega) \tag{8a}$$

where

$$\left\{\begin{matrix}\tilde{\rho}_{mn}^{e}(\gamma,\Omega)\\ \tilde{\rho}_{mn}^{o}(\gamma,\Omega)\end{matrix}\right\} = \frac{4\pi b}{a}\cos(m\Omega)\left\{\begin{matrix}\cos\left(\gamma d\cos\Omega + \frac{m\pi}{2}\right)\\ -j\sin\left(\gamma d\cos\Omega + \frac{m\pi}{2}\right)\end{matrix}\right\}$$

$$\cdot\int_0^a \rho_n(r')J_m(\gamma r')r'\,dr' \qquad (m = 0,\cdots,p;\ n = 0,\cdots,q). \tag{8b}$$

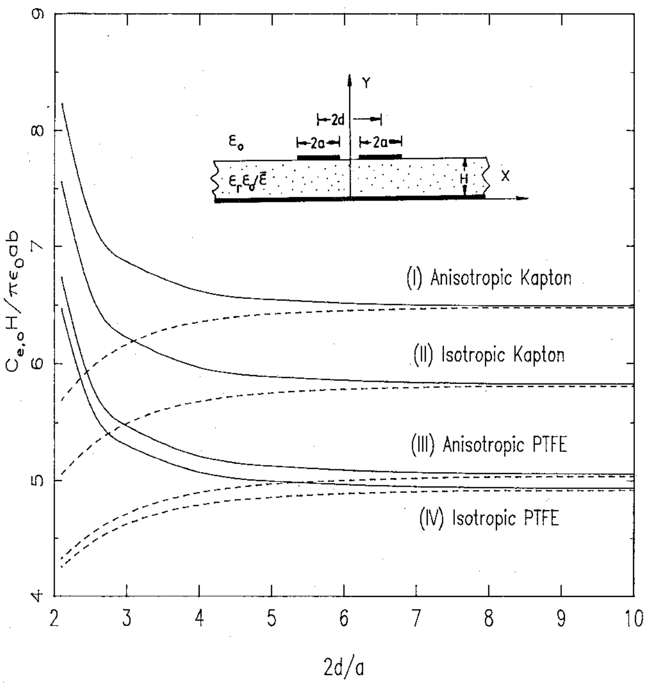

Fig. 3. Normalized modal capacitances of coupled elliptic microstrip disks for different isotropic and anisotropic substrates as a function of the normalized distance between the disks $2d/a$. Solid lines stand for the odd mode and dashed lines stand for the even mode ($a/H = 1.0$ and $b/H = 5.0$ are used in all the graphs). The permittivities are given by

(*I*) Isotropic Kapton: $\epsilon_r = 3.0$.
(*II*) Anisotropic Kapton: $\epsilon_{11}^* = \epsilon_{33}^* = 3.0$; $\epsilon_{22}^* = 3.5$; $\epsilon_{23}^* = 0$.
(*III*) Isotropic PTFE: $\epsilon_r = 2.45$.
(*IV*) Anisotroptic PTFE: $\epsilon_{11}^* = 2.89$; $\epsilon_{22}^* = 2.45$; $\epsilon_{33}^* = 2.95$; $\epsilon_{23}^* = 0$.

The integrals appearing in (8b) stand for the mth Hankel transform of the radial trial functions $\rho_n(r')$.

When the new spectral variables γ and Ω defined in (7) are used in (3) and the expansions given in (8a) for the modal charge density transforms $\tilde{\rho}_{e,o}(\gamma, \Omega)$ are introduced in the resulting expressions for $U_{e,o}$, the modal energies become explicit functions of the unknown coefficients $c_{mn}^{e,o}$ $(m = 0, \cdots, p;$ $n = 0, \cdots, q)$. To obtain these coefficients, the energy in each mode is separately required to be a minimum on condition that the absolute value of the total charge on each plate be equal to 1 [5]. Once the coefficients $c_{mn}^{e,o}$ are determined, upper bounds for the modal energies $U_{e,o}$ and lower bounds for the lower capacitances $C_{e,o}$ $(=1/U_{e,o})$ can be obtained.

III. Numerical Results

In Fig. 2, we compare our results for the gap capacitance of coupled circular microstrip disks (zero eccentricity) printed on isotropic substrates with those reported in [4]. As far as graphic comparison is possible, agreement seems to be very good.

In Fig. 3, we show how the values of the normalized modal capacitances are affected when the effect of anisotropy is neglected. When Kapton is used as a substrate, the effect of anisotropy is investigated by changing its dielectric constant in the y direction from the isotropic value to the anisotropic value. In the case of PTFE, the dielectric constants in the x and z directions are assumed to change. The difference between the isotropic and the anisotropic results is about 10% in the case of Kapton and about 2.5% in the case of PTFE.

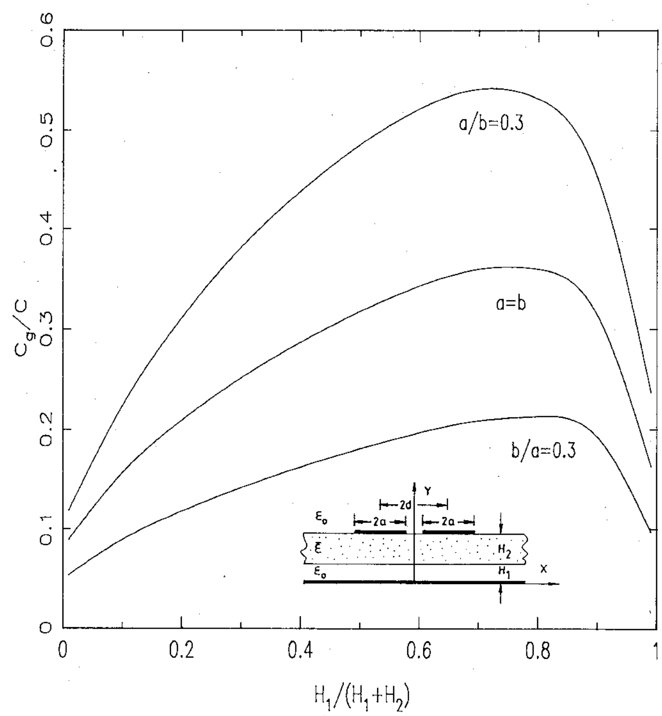

Fig. 4. Coupling coefficient C_g/C of coupled elliptic and circular microstrip disks in suspended configuration as a function of the vacuum filling fraction. The dielectric is sapphire with aligned optical axis: ($\epsilon_{11}^* = \epsilon_{33}^* = 9.4$; $\epsilon_{22}^* = 11.6$; $\epsilon_{23}^* = 0$); $a/(H_1 + H_2) = 1$; $2(d - a)/(H_1 + H_2) = 0.1$.

In Fig. 4, coupled elliptic disks on a suspended microstrip configuration are analyzed. The coupling coefficient C_g/C, which is a nondimensional parameter defined in [3], is obtained as a function of the relative position of the interface between vacuum and dielectric. Optimum coupling is reached for a certain position of that interface in which the ratio of the portion of electric energy between the disks to the portion of electric energy beneath the disks becomes a maximum.

IV. Conclusions

Variational techniques in the spectral domain are applied to the calculation of the modal capacitances and the gap capacitance of two symmetrically coupled microstrip elliptic disks. The disks are embedded in multilayered anisotropic substrates. Trial functions for the charge density on the disks are chosen in such a way that they satisfy the physical constraints of the problem.

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